SIMULATION AND SOFTWARE DEVELOPMENT FOR SOLVING INTERNAL BALLISTICS PROBLEMS

МОДЕЛИРОВАНИЕ И ПРОГРАММНОЕ ОБЕСПЕЧЕНИЕ ЗАДАЧ ВНУТРЕННЕЙ БАЛЛИСТИКИ

PhD. Stelia O., PhD. Potapenko L., eng. Sirenko I.,
Faculty of Computer Sciences and Cybernetics, Taras Shevchenko National University of Kyiv, Ukraine.
oleg.stelya@gmail.com, lpotapenko@ukr.net, i.sirenko@gmail.com

Abstract: *The problem of internal ballistics is considered, including basic pyrodynamic equation, law charge combustion, gasification law and equations of the projectile motion. A numerical method is used to solve a system of three algebraic and three ordinary differential equations. The software was developed and a number of computational experiments were made to analyze the influence of the charging parameter on the processes occurring during the projectile movement in the barrel channel. The problem of gunpowder burning and a finite-difference method to solve it are considered. The results of this work can be applied to the design of new types of trunks and charges.*
***KEYWORDS**: INTERNAL BALLISTICS, MODEL, NUMERICAL SIMULATION, PYRODYNAMIC CHARACTERISTICS.*

1. Introduction

Modern approaches based on ballistic computer models should be used to improve the efficiency of artillery defeat means. These developments are also required to create new artillery systems, shells and charges. Fundamental research in the field of ballistic theory, mathematical and cybernetic methods for solving problems of ballistics, and software development are required for this.

This paper is a continuation of works [1, 2] devoted to the development of mathematical methods, algorithms and software for solving ballistics problems.

Internal ballistics investigates the phenomena occurring in the barrel weapons under powder gases and other processes at a shot in the barrel [3–6]. Solving the problem of internal ballistics allows determining the dependence of the projectile velocity in the barrel on the time and the path which projectile passing in channel under different conditions of charging and calculating the parameters of new artillery systems.

The papers [7– 10] are dedicated to using of mathematical models of internal ballistics problems. The work [7] describes a mathematical model of shot for low pressure in space behind the projectile. The paper [8] considers a mathematical model of internal ballistics based on the physical laws of powder combustion, and experimental results obtained by burning powder in the closed vessel. In [9] the mathematical model for the weapon system with additional propelling charges along the barrel to increase muzzle velocity were developed.

Paper [11] analyzes the changing characteristics of ballistic powder charges artillery ammunition of naval artillery systems for small caliber at post-warranty stages of storage on the basis of internal ballistics model. Papers [12–14] describe sophisticated models of artillery shot, using one- and two-dimensional gas dynamic equations of transient multiphase environments. Computer model of internal ballistics is widely used not only for military applications but also for civilians. For example, in [15] internal ballistics model was used to calculate the damper in the safety grooves of the car.

The aim of this work is to develop a numerical algorithm for solving the basic problems of internal ballistics, to develop the software for pyrodynamic processes simulation depending on the charging parameters. The mathematical model of gunpowder combustion is considered too. To implement this model a finite-difference scheme was developed using parabolic spline.

The paper presents the results of calculations for influence of the powder thickness tape on the pyrodynamic characteristics. Unlike analytical methods for solving basic problems of internal ballistics [3], the computer implementation is advantageous because it allows setting parameters of the model not only as a constant, but in the form of functional relationships and considering the increasing number of factors affecting the process shot.

2. Mathematical model of internal ballistics

Mathematical model of internal ballistics consists of six equations: three differential and three algebraic ones. According to [3], the system can be written as

$$\psi = \chi z(1+\lambda z), \quad (1)$$

$$p = fw\frac{\psi - \dfrac{\theta\varphi q v^2}{2fw}}{s(l_\psi + l)}, \quad (2)$$

$$\varphi q\frac{dv}{dt} = sp, \quad (3)$$

$$\frac{dl}{dt} = v, \quad (4)$$

$$\frac{dz}{dt} = \frac{p}{I_k}, \quad (5)$$

$$l_\psi = l_0\left[1 - \frac{\Delta}{\delta} - (\alpha - \frac{1}{\delta})\Delta\psi\right]. \quad (6)$$

Equation (1) describes the formation of powder gases. Here ψ is the relative mass of burned gunpowder, z is the relative thickness of the burned powder, and χ, λ are the powder grain form factors. These values are connected by the ratio $\lambda = (1-\chi)/\chi$.

Equation (2) is the pyrodynamic basic equation that describes the process of expanding powder gases.

Here p is the pressure of powder gases, f is the force of powder, w is the mass of powder charge, θ is the expanding powder gases parameter, φ is the fictitious factor, q is the mass of the projectile, v is the projectile velocity, s is the chamber sectional area, l_ψ is the effective length of a chamber free volume, which given by (6), l_0 is the effective length of the chamber, l is the current position of the projectile in the barrel ($0 \le l \le L$, where L is the total path of the projectile in the barrel), δ is the powder density, α is the powder gases covolume, Δ is the charge density, I_k is the final pressure impulse powder gases.

The values I_k, Δ, and l_0 are calculated by formulas:

$$I_k = \frac{e_1}{u_1},\ \Delta = \frac{w}{W_0},\ l_0 = \frac{W_0}{s},$$

where e_1 is the thickness of the powder burning layer, u_1 is the powder burning speed ratio, W_0 is the chamber volume.

This system is closed with initial conditions and used to determine the pyrodynamic parameters at any time moment t.

The initial condition for equation (5) is calculated from equations (1) and (2) at the end of pyrostatic period when some powder burns. This amount of powder is determined by the values ψ_0 and z_0. We have set forcing pressure p_0 for calculate these values.

So, with (2) we have

$$p_0 = \frac{f\Delta\psi_0}{1 - \frac{\Delta}{\delta} - (\alpha - \frac{1}{\delta})\Delta\psi_0}.$$

Solving this equation with regard to ψ_0, we get

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}$$

Knowing ψ_0, we found z_0 from (1):

$$\psi_0 = \chi z_0 (1 + \lambda z_0)$$

$$z_0 = \frac{-1 + \sqrt{1 + \frac{4\lambda\psi_0}{\chi}}}{2\lambda}$$

The projectile motion starts at zero time. The system of equations (1)–(6) is solved with such initial conditions

$$v = 0,\ l = 0,\ z = z_0 \text{ when } t = 0.$$

3. *Mathematical model of stationary gunpowder combustion*

Stationary combustion of gunpowder is described of equations [16, 17]

$$D\frac{d^2T}{dx^2} - V\frac{dT}{dx} = -f,\quad x \in (0, L), \tag{7}$$

$$\frac{dy}{dx} = \Phi(T), \tag{8}$$

here T is the temperature, D is the coefficient of thermal conductivity, $V = u \cdot c \cdot \rho$, u is the speed of the front of combustion, c is the heat capacity, ρ is the density, $f = Q \cdot \Phi(T)$, Q is the heat of reaction, $\Phi(T) = k_0 \exp(-\frac{E}{RT})$, k_0 is pre-exponential factor, E is the activation energy, R is the universal gas constant, and y is the mass concentration of powder.

The appropriate boundary conditions are set for the equation (7) and initial condition is set for the equation (8).

For solving the boundary problem for the equation (7) we used the finite-difference scheme based on parabolic spline [18]. To use the finite-difference scheme it is necessary to split the interval $(0, L)$. For this purpose, two splitting (Δ_x and Δ_τ) are defined:

$$\Delta_x : 0 = x_0 < x_1 < \ldots < x_N = L, \tag{9}$$

$$\Delta_\tau : 0 = \tau_0 < \tau_1 < \ldots < \tau_{N-1} = L, \tag{10}$$

where $x_i < \tau_i < x_{i+1}, i = \overline{1, N-2}.$

Then the finire-difference scheme is written as

$$\phi_{\tau_{i-1}}(\frac{D}{h^2} + \frac{V}{h^2}\mu) - \phi_{\tau_i}(\frac{2D}{h^2} - \frac{V}{h}(1 - \frac{2\mu}{h})) + \tag{11}$$
$$+ \phi_{\tau_{i+1}}(\frac{D}{h^2} - \frac{V}{h}(1 - \frac{\mu}{h})) = -(f_{x_{i+1}} + f_{x_i})/2,\ i = \overline{1, N-2}.$$

where ϕ_{τ_i} is the lattice function that approximates the solution of equation (11) at the grid.

The finite-difference scheme is monotonous and has a unique solution. This follows from the following theorem [19].

Theorem. Let the following conditions be satisfied for the splitting (9)–(10):

$$\tau_{i+1} = \tau_i + h,\ i = \overline{0, N-2}, h > 0, N = L/h + 1,$$
$$x_1 = x_0 + h - \mu, x_{i+1} = x_i + h,\ i = \overline{1, N-2},$$
$$x_N = x_{N-1} + \mu,\ 0 < \mu < h.$$

Then, if $\mu > h - \frac{D}{|V|}$ and $V > 0$ or if $\mu < \frac{D}{|V|}$ and $V > 0$ the difference maximum principle for (11) is valid.

Since the right-hand side of Equation (7) contains the desired solution, the system of the finite-difference equations is nonlinear. The combined Newton's method and successive upper relaxation method is used for solution of the system [20]. The Kutta-Merson method is used for solving the Cauchy problem for the equation (8).

4. *The results of computational experiments*

For solving the ordinary differential equations (3)–(5) we use the Kutta-Merson method [21] . The advantage of this method over the "standard" Runge-Kutta 4th order method is an opportunity to evaluate error at each step of calculations without additional computational cost. This allows choosing a necessary step for the series of sample calculations. This method worked well for solving the problem of external ballistics [1]. The software was developed according to the constructed algorithm.

To conduct the computational experiments we should set a significant amount of numerical parameters which are divided into groups such as gunpowder parameters, gun and projectile characteristics, charging conditions.

For these calculations we used data for 100 mm guns from [22]. Computational experiments were conducted for strip of nitroglycerine gunpowder with different thickness of strip (Table 1).

We used the following gunpowder parameters: $\delta = 1600$ kg/m³, $\alpha = 0{,}001$ m³/kg, $f = 950\,000$ J/kg, $u_1 = 0{,}6 \cdot 10^{-9}$ m³/(N·s), $\theta = 0{,}2$. Charging conditions was $w = 7{,}15$ kg. Gun and projectile characteristics were $q = 16$ kg, $s = 0{,}0082$ m², $W_0 = 0{,}011$ m³, $L = 5{,}53$ m, $\varphi = 1{,}15$, $l_0 = 1{,}3415$, $p_0 = 3 \cdot 10^7$ Pa.

Examples of calculating the pyrodynamic characteristics such as dependence the pressure of powder gases in the space behind the projectile and speed of the projectile on the time are shown at Figures 1 and 2 (numbers are corresponding to different rows of Table 1). The dependence of the pressure of powder gases in space behind the projectile and speed of the projectile on the position of the projectile in the barrel is shown in Figure 3.

Using the model, varying the gunpowder strip thickness, we can adjust the rate of gas formation at a shot and get the optimal ratio between maximum pressure in the space behind the projectile and muzzle velocity. The importance of such calculations is due to the fact that the thinner powder gives more gas per unit time and may create a pressure that exceeds its allowable value for the strength of the barrel. On the other hand, the thicker powder may have not time to burn in the barrel.

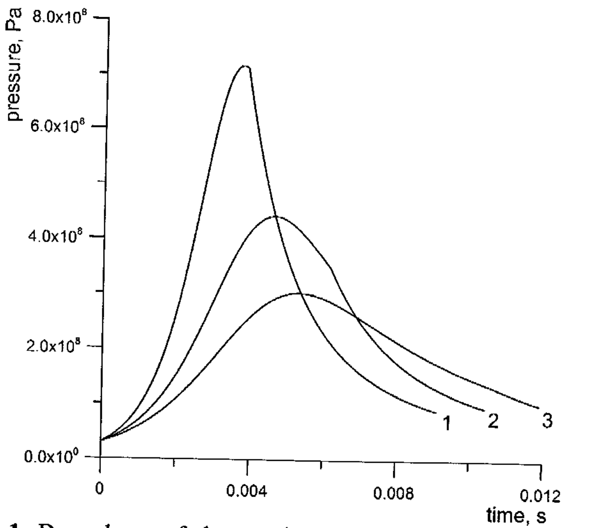

Fig. 1. Dependence of the powder gases pressure in the space behind the projectile on the time at the shot with different powder strip thicknesses

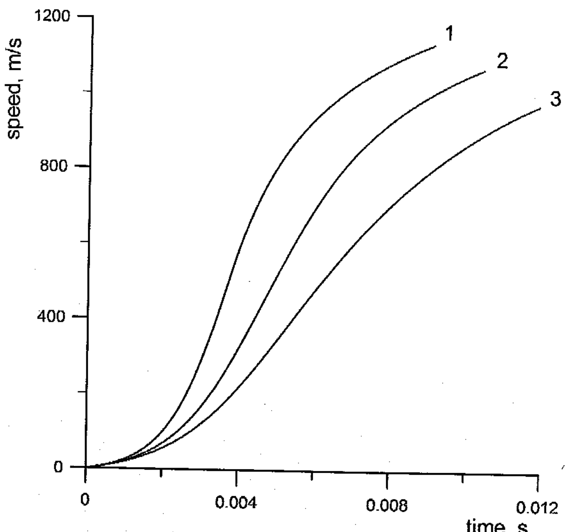

Fig. 2. Dependence of the projectile speed on the time of the shot with different powder strip thicknesses

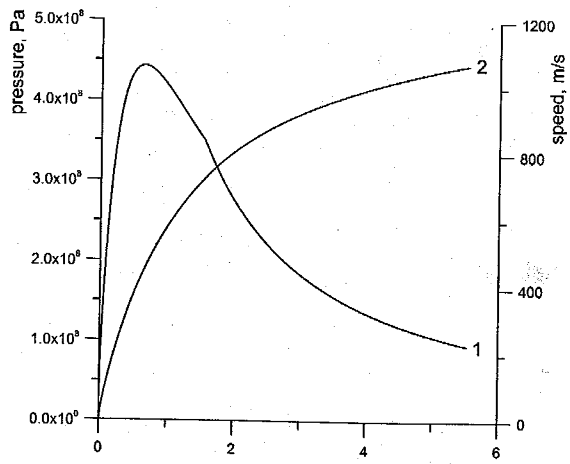

Fig. 3. Dependence of pressure powder gases (1) and the projectile speed (2) on its position in the barrel when fired (l)

Figure 4 shows pyrodynamic curves corresponding to the different thicknesses powder strips. Number of graphs corresponds to the different rows of Table 1.

The curves of Figures 1 and 4 have the points at which the derivative is ruptured. These are the points of the end of the first pyrodynamic period.

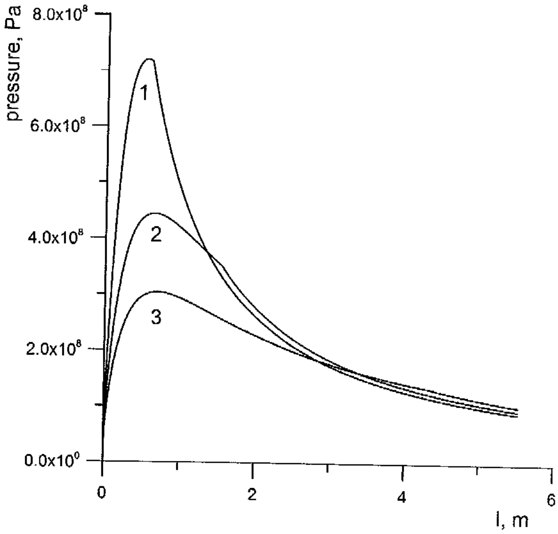

Fig. 4. Dependence of the powder gases pressure in the space behind the projectile on the position of the projectile in the barrel during firing for different powder strip thicknesses

Table 1: Value of pirodynamic characteristics for different thicknesses of powder strip

№	Strip thicknesses $2e_1$, mm	Maximum pressure, Pa	Muzzle velocity, m/s
1	1,5	$0{,}71839 \cdot 10^9$	1135,2
2	2,0	$0{,}44335 \cdot 10^9$	1070,0
3	2,5	$0{,}30337 \cdot 10^9$	974,6

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