MATHEMATICAL MODEL OF STATIONARY PROPELLANT COMBUSTION

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Stationary combustion of propellant is described of equations [1, 2]

$$D\frac{d^2T}{dx^2} - V\frac{dT}{dx} = -f, \quad x \in (0, L), \tag{1}$$

$$\frac{dy}{dx} = \Phi(T), \tag{2}$$

here T – temperature, D – coefficient of thermal conductivity, $V = u \cdot c \cdot \rho$, u – the speed of the front of combustion, c – heat capacity, ρ – density, $f = Q \cdot \Phi(T)$, Q – the heat of reaction, $\Phi(T) = k_0 \exp(-\frac{E}{RT})$, k_0 – pre-exponential factor, E – activation energy, R – the universal gas constant, y – mass concentration of propellant.

The appropriate boundary conditions set for the equation (1) and initial condition set for equation (2).

Difference scheme was used for solving the boundary problem for the equation (1) basing on parabolic spline [3]. To use difference scheme necessary to make the digitization of interval $(0, L)$. For this purpose, two splitting (Δ_x and Δ_τ) are defined:

$$\Delta_x : 0 = x_0 < x_1 < \ldots < x_N = L, \tag{3}$$

$$\Delta_\tau : 0 = \tau_0 < \tau_1 < \ldots < \tau_{N-1} = L, \tag{4}$$

here $x_i < \tau_i < x_{i+1}, i = \overline{1, N-2}.$

Then the difference scheme is written as

$$\phi_{\tau_{i-1}}(\frac{D}{h^2} + \frac{V}{h^2}\mu) - \phi_{\tau_i}(\frac{2D}{h^2} - \frac{V}{h}(1 - \frac{2\mu}{h})) +$$

$$+\phi_{\tau_{i+1}}(\frac{D}{h^2} - \frac{V}{h}(1 - \frac{\mu}{h})) = -(f_{x_{i+1}} + f_{x_i})/2, \quad i = \overline{1, N-2}. \tag{5}$$

here ϕ_{τ_i} – the lattice function that approximates the solution of equation (5) in the grid.

Difference scheme is monotonous and there it is the only solution. This follows from the following theorem [3].

Theorem. Following conditions was perfomed for the split (3)–(4):

$$\tau_{i+1} = \tau_i + h, i = \overline{0, N-2}, h > 0, N = L/h + 1,$$
$$x_1 = x_0 + h - \mu, x_{i+1} = x_i + h, i = \overline{1, N-2},$$
$$x_N = x_{N-1} + \mu, \ 0 < \mu < h.$$

Then, if $\mu > h - \frac{D}{|V|}$ and $V > 0$ or if $\mu < \frac{D}{|V|}$ and $V > 0$ for (5) maximum difference principle is performed.

Because the right side of the equation (1) contains the desired solution, the system of difference equations will be nonlinear. The combined Newton's method and successive upper relaxation method is used for solution of the system. The Kutt-Merson method is used for solving the Cauchy problem for the equation (2).

References

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