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В сборнике представлены труды VIII Московской международной конференции по исследованию операций. Конференция проводится факультетом вычислительной математики и кибернетики МГУ имени М.В. Ломоносова, Федеральным исследовательским центром «Информатика и управление» Российской академии наук (ФИЦ ИУ РАН) и Российским научным обществом исследования операций (РНОИО). На конференции обсуждаются математические вопросы исследования операций в экономике, экологии, социологии, биологии, медицине, политологии, а также численные методы исследования операций.

Ключевые слова: исследование операций; математическое моделирование; методы оптимизации.

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The conference is organized by Faculty of Computational Mathematics and Cybernetics of Lomonosov Moscow State University (MSU), Federal Research Center “Computer Science and Control” of Russian Academy of Sciences (FRC CSC RAS), and Russian Scientific Operations Research Society (RSORS), and is dedicated to the memory of an outstanding Russian scientist, full member of RAS P. S. Krasnoschekov. The conference brings together scientists from all over the world to discuss theoretical aspects and various applications of operations research. The conference aims to consider mathematical problems of operations research, latest achievements, new models in economics, ecology, medicine, political science, etc.

Keywords: operations research; mathematical modeling; optimization methods.

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References

1. Rockafellar R.T. Convex Analysis. Princeton. New York: Princeton Univ. Press. 1970.
2. Leichtweiss K. Konvexe Mengen. Berlin. 1980.

A two-step proximal algorithm of solving the problem of equilibrium programming

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Let C be a nonempty closed convex subset of a real Hilbert space H and $F : C \times C \to \mathbb{R}$ be a bifunction with $F(x, x) = 0$ for all $x \in C$. Consider the following equilibrium problem in the sense of Blum and Oettli [1, 2]:

$$\text{find} \quad x \in C \quad \text{such that} \quad F(x, y) \geq 0 \quad \forall\, y \in C.$$

We propose a new iterative two-step proximal algorithm for solving the problem of equilibrium programming in a Hilbert space. This method is a result of extension of L. D. Popov's modification of Arrow-Hurwicz scheme for approximation of saddle points of convex-concave functions [3, 4]. More precisely, we propose and analyse the following algorithm: for x_1, $y_1 \in C$ generate the sequences x_n, $y_n \in C$ with the iterative scheme

$$\begin{cases} x_{n+1} \in \operatorname{prox}_{\lambda F(y_n,\cdot)} x_n = \operatorname{argmin}_{y \in C} \left\{ \lambda F(y_n, y) + \frac{1}{2}\|y - x_n\|^2 \right\}, \\ y_{n+1} \in \operatorname{prox}_{\lambda F(y_n,\cdot)} x_{n+1} = \operatorname{argmin}_{y \in C} \left\{ \lambda F(y_n, y) + \frac{1}{2}\|y - x_{n+1}\|^2 \right\}, \end{cases}$$

where $\lambda > 0$.

The convergence of the algorithm is proved under the assumption that the solution exists and the bifunction is pseudo-monotone and Lipschitz-type.

References

1. Blum E., Oettli W. From optimization and variational inequalities to equilibrium problems // Math. Stud. 1994. V. 63. P. 123–145.
2. Combettes P.L., Hirstoaga S.A. Equilibrium programming in Hilbert spaces // Journal of Nonlinear and Convex Analysis. 2005. V. 6, № 1. P. 117–136.

3. Popov L.D. A modification of the Arrow-Hurwicz method for search of saddle points // Mathematical notes of the Academy of Sciences of the USSR. 1980. V. 28, № 5. P. 845–848.
4. Malitsky Yu.V., Semenov V.V. An Extragradient Algorithm for Monotone Variational Inequalities // Cybernetics and Systems Analysis. 2014. V. 50. P. 271–277.

Global optimality conditions for d.c. programming*

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Consider the optimization problem:

$$(\mathcal{P}): \qquad \left.\begin{array}{ll} f_0(x) \downarrow \min\limits_x, & x \in S \subset I\!R^n, \\ f_i(x) \leq 0, & i \in I := \{1, \ldots, m\}, \end{array}\right\} \tag{1}$$

where all $f_i = g_i(x) - h_i(x),\ \ i \in I \cup \{0\}$ with smooth convex functions $g_i(\cdot),\ \ h_i(\cdot),\ \ g_i, h_i :\ I\!R^n \to I\!R,\ i \in I \cup \{0\}$.

Let introduce the l_∞-penalty function [1]–[7]

$$W(x) := \max\{0, f_1(x), \ldots, f_m(x)\} = \max\{0, f_i(x), i \in I\}. \tag{2}$$

Further, consider the penalized problem as follows ($\sigma > 0$)

$$(\mathcal{P}_\sigma): \qquad \Theta_\sigma(x) := f_0(x) + \sigma W(x) \downarrow \min_x, \quad x \in S. \tag{3}$$

As well-known [1]–[7], if $z \in Sol(\mathcal{P}_\sigma)$, and $z \in D := \{x \in S :\ f_i(x) \leq 0,\ i \in I\}$, then $z \in Sol(\mathcal{P})$. In addition, if $z \in Sol(\mathcal{P})$, then under supplementary conditions [2, 3, 5, 7] for some $\sigma_* > 0$, $\sigma_* \geq \| \lambda_z \|_1$ (where λ_z is the KKT-multiplier corresponding to z), the inclusion $z \in Sol(\mathcal{P}_\sigma)$ holds. Moreover [6], $Sol(\mathcal{P}) = Sol(\mathcal{P}_\sigma)$, so that Problems $(\mathcal{P})$ and $(\mathcal{P}_\sigma)$ turn out to be equivalent $\forall \sigma \geq \sigma_*$.

It can be readily seen that the penalized function $\Theta_\sigma(\cdot)$ is a d.c. function, since the functions $f_i(\cdot),\ \ i \in I \cup \{0\}$, are as such. Actually, since $\sigma > 0$,

$$\Theta_\sigma(x) = G_\sigma(x) - H_\sigma(x), \tag{4}$$

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